5.1. Neodređeni integral

11. 12. 2020.

Fotografija: Jeremy Thomas

Definicija

Neka je $f : I \to \mathbb{R}$, gdje je I otvoren interval u $\mathbb{R}$. **Antiderivacija** ili **primitivna funkcija** od f je svaka funkcija $F : I \to \mathbb{R}$ takva da vrijedi

$$F'(x) = f(x) \qquad \text{za sve } x \in I.$$

Neodređen integral od f je skup svih njenih antiderivacija. Oznaka:

$$\int f(x)\,\mathrm{d}x.$$

Svojstva antiderivacije i neodređenog integrala

- Nema svaka funkcija $f : I \to \mathbb{R}$ antiderivaciju, ali svaka neprekidna funkcija $f : I \to \mathbb{R}$ ima.

Svojstva antiderivacije i neodređenog integrala

- Nema svaka funkcija $f : I \to \mathbb{R}$ antiderivaciju, ali svaka neprekidna funkcija $f : I \to \mathbb{R}$ ima.
- Ako je F jedna antiderivacija od f, tada je

$$\int f(x)\,\mathrm{d}x = \{F + C : C \in \mathbb{R}\}.$$

  Kraće pišemo

$$\int f(x)\,\mathrm{d}x = F(x) + C, \quad C \in \mathbb{R}.$$

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- Neka je $a \in \mathbb{R}$. Sljedeće jednakosti vrijede kad god je njihova desna strana definirana:
  (i) $\int a\,f(x)\,\mathrm{d}x = a \int f(x)\,\mathrm{d}x$
  (ii) $\int (f(x) \pm g(x))\,\mathrm{d}x = \int f(x)\,\mathrm{d}x \pm \int g(x)\,\mathrm{d}x.$

Tablica integrala

$$\int \mathrm{d}x = x + C$$

$$\int x^a \,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

$$\int e^x \,\mathrm{d}x = e^x + C$$

$$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x \,\mathrm{d}x = \sin x + C$$

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$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

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Zašto je $\int \frac{\mathrm{d}x}{x} = \ln|x| + C$?

- Zašto ne
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Zato što je $\mathcal{D}_{\frac{1}{x}} = \mathbb{R} \setminus \{0\}$, a $\mathcal{D}_{\ln x} = \langle 0, +\infty \rangle$.
- S druge strane, $\mathcal{D}_{\ln|x|} = \mathbb{R} \setminus \{0\} = \mathcal{D}_{\frac{1}{x}}$ i, s obzirom da je
$$|x| = \begin{cases} x, & \text{ako je } x \geq 0, \\ -x, & \text{ako je } x < 0, \end{cases} \qquad \text{pa je} \qquad \ln|x| = \begin{cases} \ln x, & \text{ako je } x > 0, \\ \ln(-x), & \text{ako je } x < 0, \end{cases}$$
imamo
$$(\ln|x|)' = \begin{cases} (\ln x)' = \frac{1}{x}, & \text{ako je } x > 0, \\ (\ln(-x))' = \frac{1}{-x} \cdot (-1) = \frac{1}{x}, & \text{ako je } x < 0, \end{cases}$$
dakle
$$(\ln|x|)' = \frac{1}{x}, \qquad x \in \mathbb{R} \setminus \{0\},$$
što znači da je $\ln|x|$ antiderivacija funkcije $\frac{1}{x}$.

Zadatak 45(a)

Izračunajte integral

$$\int x^5 \, \mathrm{d}x.$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

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Zadatak 45(a)

Izračunajte integral

$$\int x^5 \, \mathrm{d}x.$$

Rješenje. Imamo

$$\int x^5 \, \mathrm{d}x = \frac{x^6}{6} + C.$$

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Zadatak 45(b)

Izračunajte integral

$$\int \frac{1}{x^{\frac{1}{4}}}\, \mathrm{d}x.$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

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$$\int \frac{1}{x^{\frac{1}{4}}}\,\mathrm{d}x = \int x^{-\frac{1}{4}}\,\mathrm{d}x$$

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Rješenje. Imamo
$$\int \frac{1}{x^{\frac{1}{4}}}\,\mathrm{d}x = \int x^{-\frac{1}{4}}\,\mathrm{d}x$$
$$= \frac{x^{\frac{3}{4}}}{\frac{3}{4}} + C$$

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$$\int \frac{1}{x^{\frac{1}{4}}}\,\mathrm{d}x = \int x^{-\frac{1}{4}}\,\mathrm{d}x$$
$$= \frac{x^{\frac{3}{4}}}{\frac{3}{4}} + C$$
$$= \frac{4}{3}x^{\frac{3}{4}} + C.$$

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Zadatak 45(c)

Izračunajte integral

$$\int \sqrt{x \cdot \sqrt[3]{x}}\, \mathrm{d}x.$$

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Izračunajte integral

$$\int \sqrt{x \cdot \sqrt[3]{x}}\,\mathrm{d}x.$$

Rješenje.

$$\int \sqrt{x \cdot \sqrt[3]{x}}\,\mathrm{d}x = \int \left(x^1 \cdot x^{\frac{1}{3}}\right)^{\frac{1}{2}}\,\mathrm{d}x$$

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Rješenje.
$$\int \sqrt{x \cdot \sqrt[3]{x}}\,\mathrm{d}x = \int \left(x^{1} \cdot x^{\frac{1}{3}}\right)^{\frac{1}{2}}\,\mathrm{d}x$$
$$= \int \left(x^{1+\frac{1}{3}}\right)^{\frac{1}{2}}\,\mathrm{d}x$$
$$= \int x^{\frac{4}{3}\cdot\frac{1}{2}}\,\mathrm{d}x$$

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Zadatak 45(c)

Izračunajte integral

$$\int \sqrt{x \cdot \sqrt[3]{x}}\,\mathrm{d}x.$$

Rješenje.

$$\begin{aligned}\int \sqrt{x \cdot \sqrt[3]{x}}\,\mathrm{d}x &= \int \left(x^1 \cdot x^{\frac{1}{3}}\right)^{\frac{1}{2}} \mathrm{d}x \\ &= \int \left(x^{1+\frac{1}{3}}\right)^{\frac{1}{2}} \mathrm{d}x \\ &= \int x^{\frac{4}{3}\cdot\frac{1}{2}}\,\mathrm{d}x \\ &= \int x^{\frac{2}{3}}\,\mathrm{d}x\end{aligned}$$

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Zadatak 45(c)

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Rješenje.

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\int \sqrt{x \cdot \sqrt[3]{x}}\,\mathrm{d}x &= \int \left(x^1 \cdot x^{\frac{1}{3}}\right)^{\frac{1}{2}} \mathrm{d}x \\
&= \int \left(x^{1+\frac{1}{3}}\right)^{\frac{1}{2}} \mathrm{d}x \\
&= \int x^{\frac{4}{3}\cdot\frac{1}{2}}\,\mathrm{d}x \\
&= \int x^{\frac{2}{3}}\,\mathrm{d}x \\
&= \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + C
\end{aligned}$$

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Zadatak 45(c)

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$$\int \sqrt{x \cdot \sqrt[3]{x}}\, \mathrm{d}x.$$

Rješenje.

$$\begin{aligned}
\int \sqrt{x \cdot \sqrt[3]{x}}\, \mathrm{d}x &= \int \left(x^1 \cdot x^{\frac{1}{3}}\right)^{\frac{1}{2}} \mathrm{d}x \\
&= \int \left(x^{1+\frac{1}{3}}\right)^{\frac{1}{2}} \mathrm{d}x \\
&= \int x^{\frac{4}{3}\cdot\frac{1}{2}}\, \mathrm{d}x \\
&= \int x^{\frac{2}{3}}\, \mathrm{d}x \\
&= \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + C \\
&= \frac{3}{5}x^{\frac{5}{3}} + C.
\end{aligned}$$

$$\int \mathrm{d}x = x + C$$

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Zadatak 45(d)

Izračunajte integral

$$\int \left(3x^4 + 4x^{-5}\right)\, \mathrm{d}x.$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a \,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x \,\mathrm{d}x = e^x + C$$
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$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(d)

Izračunajte integral

$$\int \left(3x^4 + 4x^{-5}\right)\, \mathrm{d}x.$$

Rješenje. Imamo

$$\int \left(3x^4 + 4x^{-5}\right)\, \mathrm{d}x = 3 \cdot \frac{x^5}{5} + 4 \cdot \frac{x^{-4}}{-4} + C$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\, \mathrm{d}x = e^x + C$$

$$\int a^x\, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\, \mathrm{d}x = \sin x + C$$

$$\int \sin x\, \mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(d)

Izračunajte integral

$$\int \left(3x^4 + 4x^{-5}\right)\, \mathrm{d}x.$$

Rješenje. Imamo

$$\begin{aligned}\int \left(3x^4 + 4x^{-5}\right)\, \mathrm{d}x &= 3 \cdot \frac{x^5}{5} + 4 \cdot \frac{x^{-4}}{-4} + C \\ &= \frac{3}{5}x^5 - x^{-4} + C.\end{aligned}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\, \mathrm{d}x = e^x + C$$

$$\int a^x\, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\, \mathrm{d}x = \sin x + C$$

$$\int \sin x\, \mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1 - x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1 + x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(e)

Izračunajte integral

$$\int \frac{x - 4x^3 + 2x^{\frac{1}{3}}}{x^{\frac{4}{5}}}\,\mathrm{d}x.$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(e)

Izračunajte integral

$$\int \frac{x - 4x^3 + 2x^{\frac{1}{3}}}{x^{\frac{4}{5}}}\,\mathrm{d}x.$$

Rješenje. Imamo

$$\int \frac{x - 4x^3 + 2x^{\frac{1}{3}}}{x^{\frac{4}{5}}}\,\mathrm{d}x = \int \left(x^{\frac{1}{5}} - 4x^{\frac{11}{5}} + 2x^{-\frac{7}{15}}\right)\mathrm{d}x$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1 - x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1 + x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Izračunajte integral

$$\int \frac{x - 4x^3 + 2x^{\frac{1}{3}}}{x^{\frac{4}{5}}}\,\mathrm{d}x.$$

Rješenje. Imamo

$$\int \frac{x - 4x^3 + 2x^{\frac{1}{3}}}{x^{\frac{4}{5}}}\,\mathrm{d}x = \int \left(x^{\frac{1}{5}} - 4x^{\frac{11}{5}} + 2x^{-\frac{7}{15}}\right)\,\mathrm{d}x$$

$$= \frac{x^{\frac{6}{5}}}{\frac{6}{5}} - 4 \cdot \frac{x^{\frac{16}{5}}}{\frac{16}{5}} + 2 \cdot \frac{x^{\frac{8}{15}}}{\frac{8}{15}} + C$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(e)

Izračunajte integral

$$\int \frac{x - 4x^3 + 2x^{\frac{1}{3}}}{x^{\frac{4}{5}}}\,\mathrm{d}x.$$

Rješenje. Imamo

$$\begin{aligned}\int \frac{x - 4x^3 + 2x^{\frac{1}{3}}}{x^{\frac{4}{5}}}\,\mathrm{d}x &= \int \left(x^{\frac{1}{5}} - 4x^{\frac{11}{5}} + 2x^{-\frac{7}{15}}\right)\,\mathrm{d}x \\ &= \frac{x^{\frac{6}{5}}}{\frac{6}{5}} - 4\cdot\frac{x^{\frac{16}{5}}}{\frac{16}{5}} + 2\cdot\frac{x^{\frac{8}{15}}}{\frac{8}{15}} + C \\ &= \frac{5}{6}x^{\frac{6}{5}} - \frac{5}{4}x^{\frac{16}{5}} + \frac{15}{4}x^{\frac{8}{15}} + C.\end{aligned}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(f)

Izračunajte integral

$$\int 5^x \,\mathrm{d}x.$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \,\mathrm{d}x = e^x + C$$

$$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x \,\mathrm{d}x = \sin x + C$$

$$\int \sin x \,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(f)

Izračunajte integral

$$\int 5^x \, \mathrm{d}x.$$

Rješenje. Imamo

$$\int 5^x \, \mathrm{d}x = \frac{5^x}{\ln 5} + C.$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \, \mathrm{d}x = e^x + C$$

$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0, \ a \neq 1)$$

$$\int \cos x \, \mathrm{d}x = \sin x + C$$

$$\int \sin x \, \mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(g)

Izračunajte integral

$$\int (6\sin x + 5\cos x)\,\mathrm{d}x.$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(g)

Izračunajte integral

$$\int (6\sin x + 5\cos x)\,\mathrm{d}x.$$

Rješenje. Imamo

$$\int (6\sin x + 5\cos x)\,\mathrm{d}x = -6\cos x + 5\sin x + C.$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(h)

Izračunajte integral

$$\int \mathrm{tg}^2 x \, \mathrm{d}x.$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \, \mathrm{d}x = e^x + C$$

$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x \, \mathrm{d}x = \sin x + C$$

$$\int \sin x \, \mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \mathrm{tg}\, x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\mathrm{ctg}\, x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \mathrm{arctg}\, x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \mathrm{arctg}\, \frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(h)

Izračunajte integral

$$\int \operatorname{tg}^2 x \, \mathrm{d}x.$$

Rješenje. Imamo

$$\int \operatorname{tg}^2 x \, \mathrm{d}x = \int \frac{\sin^2 x}{\cos^2 x} \, \mathrm{d}x$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \, \mathrm{d}x = e^x + C$$

$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x \, \mathrm{d}x = \sin x + C$$

$$\int \sin x \, \mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(h)

Izračunajte integral

$$\int \operatorname{tg}^2 x \, dx.$$

Rješenje. Imamo

$$\int \operatorname{tg}^2 x \, dx = \int \frac{\sin^2 x}{\cos^2 x} \, dx$$

$$= \int \frac{1 - \cos^2 x}{\cos^2 x} \, dx$$

$$\int dx = x + C$$

$$\int x^a \, dx = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{dx}{x} = \ln |x| + C$$

$$\int e^x \, dx = e^x + C$$

$$\int a^x \, dx = \frac{a^x}{\ln a} + C \quad (a > 0, \ a \neq 1)$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \frac{dx}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{dx}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{dx}{\sqrt{1 - x^2}} = \arcsin x + C$$

$$\int \frac{dx}{1 + x^2} = \operatorname{arctg} x + C$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(h)

Izračunajte integral

$$\int \operatorname{tg}^2 x \, \mathrm{d}x.$$

Rješenje. Imamo

$$\begin{aligned}\int \operatorname{tg}^2 x \, \mathrm{d}x &= \int \frac{\sin^2 x}{\cos^2 x} \, \mathrm{d}x \\ &= \int \frac{1 - \cos^2 x}{\cos^2 x} \, \mathrm{d}x \\ &= \int \left(\frac{1}{\cos^2 x} - 1 \right) \mathrm{d}x\end{aligned}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \, \mathrm{d}x = e^x + C$$

$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x \, \mathrm{d}x = \sin x + C$$

$$\int \sin x \, \mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1 - x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1 + x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(h)

Izračunajte integral

$$\int \operatorname{tg}^2 x \, \mathrm{d}x.$$

Rješenje. Imamo

$$\begin{aligned}\int \operatorname{tg}^2 x \, \mathrm{d}x &= \int \frac{\sin^2 x}{\cos^2 x} \, \mathrm{d}x \\ &= \int \frac{1 - \cos^2 x}{\cos^2 x} \, \mathrm{d}x \\ &= \int \left(\frac{1}{\cos^2 x} - 1 \right) \mathrm{d}x \\ &= \operatorname{tg} x - x + C.\end{aligned}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln |x| + C$$

$$\int e^x \, \mathrm{d}x = e^x + C$$

$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0, \ a \neq 1)$$

$$\int \cos x \, \mathrm{d}x = \sin x + C$$

$$\int \sin x \, \mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1 - x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1 + x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(i)

Izračunajte integral

$$\int \frac{1+\cos(2x)}{\cos^2 x - 1}\,\mathrm{d}x.$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(i)

Izračunajte integral

$$\int \frac{1+\cos(2x)}{\cos^2 x - 1}\,\mathrm{d}x.$$

Rješenje. Imamo

$$\int \frac{1+\cos(2x)}{\cos^2 x - 1}\,\mathrm{d}x = \int \frac{1+\cos(2x)}{-\left(1-\cos^2 x\right)}\,\mathrm{d}x$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(i)

Izračunajte integral

$$\int \frac{1+\cos(2x)}{\cos^2 x - 1}\,\mathrm{d}x.$$

Rješenje. Imamo

$$\int \frac{1+\cos(2x)}{\cos^2 x - 1}\,\mathrm{d}x = \int \frac{1+\cos(2x)}{-(1-\cos^2 x)}\,\mathrm{d}x$$

$$= \int \frac{1+\cos(2x)}{-\sin^2 x}\,\mathrm{d}x$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(i)

Izračunajte integral

$$\int \frac{1+\cos(2x)}{\cos^2 x - 1}\,\mathrm{d}x.$$

Rješenje. Imamo

$$\begin{aligned}
\int \frac{1+\cos(2x)}{\cos^2 x - 1}\,\mathrm{d}x &= \int \frac{1+\cos(2x)}{-(1-\cos^2 x)}\,\mathrm{d}x \\
&= \int \frac{1+\cos(2x)}{-\sin^2 x}\,\mathrm{d}x \\
&= \int \frac{1+\left(1-2\sin^2 x\right)}{-\sin^2 x}\,\mathrm{d}x
\end{aligned}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(i)

Izračunajte integral

$$\int \frac{1+\cos(2x)}{\cos^2 x - 1}\,\mathrm{d}x.$$

Rješenje. Imamo

$$\begin{aligned}
\int \frac{1+\cos(2x)}{\cos^2 x - 1}\,\mathrm{d}x &= \int \frac{1+\cos(2x)}{-(1-\cos^2 x)}\,\mathrm{d}x \\
&= \int \frac{1+\cos(2x)}{-\sin^2 x}\,\mathrm{d}x \\
&= \int \frac{1+\left(1-2\sin^2 x\right)}{-\sin^2 x}\,\mathrm{d}x \\
&= \int \left(-\frac{2}{\sin^2 x}+2\right)\,\mathrm{d}x
\end{aligned}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

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$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(i)

Izračunajte integral

$$\int \frac{1+\cos(2x)}{\cos^2 x - 1}\,\mathrm{d}x.$$

Rješenje. Imamo

$$\begin{aligned}
\int \frac{1+\cos(2x)}{\cos^2 x - 1}\,\mathrm{d}x &= \int \frac{1+\cos(2x)}{-(1-\cos^2 x)}\,\mathrm{d}x \\
&= \int \frac{1+\cos(2x)}{-\sin^2 x}\,\mathrm{d}x \\
&= \int \frac{1+\left(1-2\sin^2 x\right)}{-\sin^2 x}\,\mathrm{d}x \\
&= \int \left(-\frac{2}{\sin^2 x}+2\right)\,\mathrm{d}x \\
&= (-2)(-\operatorname{ctg} x) + 2x + C
\end{aligned}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

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$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(i)

Izračunajte integral
$$\int \frac{1+\cos(2x)}{\cos^2 x - 1}\,\mathrm{d}x.$$

Rješenje. Imamo
$$\begin{aligned}
\int \frac{1+\cos(2x)}{\cos^2 x - 1}\,\mathrm{d}x &= \int \frac{1+\cos(2x)}{-(1-\cos^2 x)}\,\mathrm{d}x \\
&= \int \frac{1+\cos(2x)}{-\sin^2 x}\,\mathrm{d}x \\
&= \int \frac{1+\left(1-2\sin^2 x\right)}{-\sin^2 x}\,\mathrm{d}x \\
&= \int \left(-\frac{2}{\sin^2 x}+2\right)\mathrm{d}x \\
&= (-2)(-\operatorname{ctg} x) + 2x + C \\
&= 2\operatorname{ctg} x + 2x + C.
\end{aligned}$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
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$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(j)

Izračunajte integral

$$\int \frac{\mathrm{d}x}{\sqrt{5-5x^2}}.$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \,\mathrm{d}x = e^x + C$$

$$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x \,\mathrm{d}x = \sin x + C$$

$$\int \sin x \,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(j)

Izračunajte integral

$$\int \frac{\mathrm{d}x}{\sqrt{5-5x^2}}.$$

Rješenje. Imamo

$$\int \frac{\mathrm{d}x}{\sqrt{5-5x^2}}\,\mathrm{d}x = \int \frac{\mathrm{d}x}{\sqrt{5\left(1-x^2\right)}}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

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$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(j)

Izračunajte integral

$$\int \frac{\mathrm{d}x}{\sqrt{5-5x^2}}.$$

Rješenje. Imamo

$$\int \frac{\mathrm{d}x}{\sqrt{5-5x^2}}\,\mathrm{d}x = \int \frac{\mathrm{d}x}{\sqrt{5\,(1-x^2)}} = \frac{1}{\sqrt{5}} \int \frac{\mathrm{d}x}{\sqrt{1-x^2}}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \,\mathrm{d}x = e^x + C$$

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$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(j)

Izračunajte integral

$$\int \frac{\mathrm{d}x}{\sqrt{5-5x^2}}.$$

Rješenje. Imamo

$$\begin{aligned}\int \frac{\mathrm{d}x}{\sqrt{5-5x^2}}\,\mathrm{d}x &= \int \frac{\mathrm{d}x}{\sqrt{5\,(1-x^2)}}\\ &= \frac{1}{\sqrt{5}}\int \frac{\mathrm{d}x}{\sqrt{1-x^2}}\\ &= \frac{1}{\sqrt{5}}\arcsin x + C.\end{aligned}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

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$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(k)

Izračunajte integral

$$\int \frac{x^2}{x^2+1}\,\mathrm{d}x.$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

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$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(k)

Izračunajte integral

$$\int \frac{x^2}{x^2+1}\,\mathrm{d}x.$$

Rješenje. Imamo

$$\int \frac{x^2}{x^2+1}\,\mathrm{d}x = \int \frac{\left(x^2+1\right)-1}{x^2+1}\,\mathrm{d}x$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(k)

Izračunajte integral

$$\int \frac{x^2}{x^2+1}\,\mathrm{d}x.$$

Rješenje. Imamo

$$\int \frac{x^2}{x^2+1}\,\mathrm{d}x = \int \frac{(x^2+1)-1}{x^2+1}\,\mathrm{d}x$$
$$= \int \left(1 - \frac{1}{x^2+1}\right)\mathrm{d}x$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(k)

Izračunajte integral

$$\int \frac{x^2}{x^2+1}\,\mathrm{d}x.$$

Rješenje. Imamo

$$\begin{aligned}\int \frac{x^2}{x^2+1}\,\mathrm{d}x &= \int \frac{\left(x^2+1\right)-1}{x^2+1}\,\mathrm{d}x \\ &= \int \left(1-\frac{1}{x^2+1}\right)\mathrm{d}x \\ &= x - \operatorname{arctg} x + C.\end{aligned}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

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$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(I)

Izračunajte integral

$$\int \frac{3 \cdot 2^x - 2 \cdot 3^x}{2^x} \, \mathrm{d}x.$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \, \mathrm{d}x = e^x + C$$

$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0, \ a \neq 1)$$

$$\int \cos x \, \mathrm{d}x = \sin x + C$$

$$\int \sin x \, \mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(I)

Izračunajte integral

$$\int \frac{3 \cdot 2^x - 2 \cdot 3^x}{2^x}\,\mathrm{d}x.$$

Rješenje. Imamo

$$\int \frac{3 \cdot 2^x - 2 \cdot 3^x}{2^x}\,\mathrm{d}x = \int \left(3 - 2 \cdot \left(\frac{3}{2}\right)^x\right)\mathrm{d}x$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(I)

Izračunajte integral

$$\int \frac{3 \cdot 2^x - 2 \cdot 3^x}{2^x} \, \mathrm{d}x.$$

Rješenje. Imamo

$$\int \frac{3 \cdot 2^x - 2 \cdot 3^x}{2^x} \, \mathrm{d}x = \int \left(3 - 2 \cdot \left(\frac{3}{2}\right)^x\right) \mathrm{d}x$$

$$= 3x - 2 \cdot \frac{\left(\frac{3}{2}\right)^x}{\ln \frac{3}{2}} + C.$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \, \mathrm{d}x = e^x + C$$

$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0, \; a \neq 1)$$

$$\int \cos x \, \mathrm{d}x = \sin x + C$$

$$\int \sin x \, \mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1 - x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1 + x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(m)

Izračunajte integral

$$\int \left(\frac{\sqrt{x^7} - 3 + x}{x^2} - (2^x + 5^x)^2 \right) \mathrm{d}x.$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a \,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x \,\mathrm{d}x = e^x + C$$
$$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x \,\mathrm{d}x = \sin x + C$$
$$\int \sin x \,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(m)

Izračunajte integral

$$\int \left(\frac{\sqrt{x^7} - 3 + x}{x^2} - (2^x + 5^x)^2 \right) \mathrm{d}x.$$

Rješenje. Imamo

$$\int \left(\frac{\sqrt{x^7} - 3 + x}{x^2} - (2^x + 5^x)^2 \right) \mathrm{d}x$$
$$= \int \left(x^{\frac{3}{2}} - 3x^{-2} + \frac{1}{x} - 4^x - 2 \cdot 10^x - 25^x \right) \mathrm{d}x$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x \, \mathrm{d}x = e^x + C$$
$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0, \ a \neq 1)$$
$$\int \cos x \, \mathrm{d}x = \sin x + C$$
$$\int \sin x \, \mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1 - x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1 + x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(m)

Izračunajte integral

$$\int \left(\frac{\sqrt{x^7} - 3 + x}{x^2} - (2^x + 5^x)^2 \right) \mathrm{d}x.$$

Rješenje. Imamo

$$\int \left(\frac{\sqrt{x^7} - 3 + x}{x^2} - (2^x + 5^x)^2 \right) \mathrm{d}x$$

$$= \int \left(x^{\frac{3}{2}} - 3x^{-2} + \frac{1}{x} - 4^x - 2 \cdot 10^x - 25^x \right) \mathrm{d}x$$

$$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} - 3 \cdot \frac{x^{-1}}{-1} + \ln|x| - \frac{4^x}{\ln 4} - 2 \cdot \frac{10^x}{\ln 10} - \frac{25^x}{\ln 25} + C.$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \,\mathrm{d}x = e^x + C$$

$$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x \,\mathrm{d}x = \sin x + C$$

$$\int \sin x \,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(n)

Izračunajte integral $\int \left(\left(1 - \frac{1}{x^2} \right) \sqrt{x\sqrt{x}} + 4 \sin x \right) \mathrm{d}x.$

$$\int \mathrm{d}x = x + C$$
$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x \, \mathrm{d}x = e^x + C$$
$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0, \ a \neq 1)$$
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$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1 - x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1 + x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(n)

Izračunajte integral $\displaystyle\int \left(\left(1 - \frac{1}{x^2}\right) \sqrt{x\sqrt{x}} + 4\sin x \right) \mathrm{d}x.$

Rješenje. Kako je za sve $x \in [0, +\infty\rangle$

$$\sqrt{x\sqrt{x}} = \left(x^1 \cdot x^{\frac{1}{2}}\right)^{\frac{1}{2}}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \,\mathrm{d}x = e^x + C$$

$$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

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$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(n)

Izračunajte integral $\displaystyle\int \left(\left(1-\frac{1}{x^2}\right)\sqrt{x\sqrt{x}}+4\sin x\right)\mathrm{d}x.$

Rješenje. Kako je za sve $x \in [0, +\infty\rangle$

$$\sqrt{x\sqrt{x}} = \left(x^1 \cdot x^{\frac{1}{2}}\right)^{\frac{1}{2}} = \left(x^{1+\frac{1}{2}}\right)^{\frac{1}{2}}$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a \,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x \,\mathrm{d}x = e^x + C$$
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$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(n)

Izračunajte integral $\displaystyle\int \left(\left(1-\frac{1}{x^2}\right)\sqrt{x\sqrt{x}}+4\sin x\right)\mathrm{d}x.$

Rješenje. Kako je za sve $x \in [0,+\infty\rangle$

$$\sqrt{x\sqrt{x}} = \left(x^1 \cdot x^{\frac{1}{2}}\right)^{\frac{1}{2}} = \left(x^{1+\frac{1}{2}}\right)^{\frac{1}{2}} = x^{\frac{3}{2}\cdot\frac{1}{2}}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

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$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 45(n)

Izračunajte integral $\displaystyle\int \left(\left(1-\frac{1}{x^2}\right)\sqrt{x\sqrt{x}}+4\sin x\right)\mathrm{d}x.$

Rješenje. Kako je za sve $x\in[0,+\infty\rangle$

$$\sqrt{x\sqrt{x}}=\left(x^1\cdot x^{\frac{1}{2}}\right)^{\frac{1}{2}}=\left(x^{1+\frac{1}{2}}\right)^{\frac{1}{2}}=x^{\frac{3}{2}\cdot\frac{1}{2}}=x^{\frac{3}{4}},$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1}+C \quad (a\neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a}+C \quad (a>0,\ a\neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
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$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a>0)$$

Zadatak 45(n)

Izračunajte integral $\displaystyle\int \left(\left(1-\frac{1}{x^2}\right)\sqrt{x\sqrt{x}}+4\sin x\right)\,\mathrm{d}x$.

Rješenje. Kako je za sve $x \in [0,+\infty\rangle$

$$\sqrt{x\sqrt{x}} = \left(x^1 \cdot x^{\frac{1}{2}}\right)^{\frac{1}{2}} = \left(x^{1+\frac{1}{2}}\right)^{\frac{1}{2}} = x^{\frac{3}{2}\cdot\frac{1}{2}} = x^{\frac{3}{4}},$$

imamo

$$\int \left(\left(1-\frac{1}{x^2}\right)\sqrt{x\sqrt{x}}+4\sin x\right)\,\mathrm{d}x$$

$$= \int \left(\left(1-\frac{1}{x^2}\right)\cdot x^{\frac{3}{4}}+4\sin x\right)\,\mathrm{d}x$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

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$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a>0)$$

Zadatak 45(n)

Izračunajte integral $\displaystyle\int \left(\left(1 - \frac{1}{x^2}\right) \sqrt{x\sqrt{x}} + 4\sin x \right) \mathrm{d}x.$

Rješenje. Kako je za sve $x \in [0, +\infty\rangle$

$$\sqrt{x\sqrt{x}} = \left(x^1 \cdot x^{\frac{1}{2}}\right)^{\frac{1}{2}} = \left(x^{1+\frac{1}{2}}\right)^{\frac{1}{2}} = x^{\frac{3}{2}\cdot\frac{1}{2}} = x^{\frac{3}{4}},$$

imamo

$$\begin{aligned}
&\int \left(\left(1 - \frac{1}{x^2}\right) \sqrt{x\sqrt{x}} + 4\sin x \right) \mathrm{d}x \\
&= \int \left(\left(1 - \frac{1}{x^2}\right) \cdot x^{\frac{3}{4}} + 4\sin x \right) \mathrm{d}x \\
&= \int \left(x^{\frac{3}{4}} - x^{-\frac{5}{4}} + 4\sin x \right) \mathrm{d}x
\end{aligned}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

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$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

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Zadatak 45(n)

Izračunajte integral $\displaystyle\int \left(\left(1-\frac{1}{x^2}\right)\sqrt{x\sqrt{x}}+4\sin x\right)\,\mathrm{d}x.$

Rješenje. Kako je za sve $x\in[0,+\infty\rangle$

$$\sqrt{x\sqrt{x}}=\left(x^1\cdot x^{\frac12}\right)^{\frac12}=\left(x^{1+\frac12}\right)^{\frac12}=x^{\frac32\cdot\frac12}=x^{\frac34},$$

imamo

$$\begin{aligned}
&\int \left(\left(1-\frac{1}{x^2}\right)\sqrt{x\sqrt{x}}+4\sin x\right)\,\mathrm{d}x\\
&=\int \left(\left(1-\frac{1}{x^2}\right)\cdot x^{\frac34}+4\sin x\right)\,\mathrm{d}x\\
&=\int \left(x^{\frac34}-x^{-\frac54}+4\sin x\right)\,\mathrm{d}x\\
&=\frac{x^{\frac74}}{\frac74}-\frac{x^{-\frac14}}{-\frac14}+4(-\cos x)+C.
\end{aligned}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x=\frac{x^{a+1}}{a+1}+C \quad (a\neq -1)$$

$$\int \frac{\mathrm{d}x}{x}=\ln|x|+C$$

$$\int e^x\,\mathrm{d}x=e^x+C$$

$$\int a^x\,\mathrm{d}x=\frac{a^x}{\ln a}+C \quad (a>0,\ a\neq 1)$$

$$\int \cos x\,\mathrm{d}x=\sin x+C$$

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$$\int \frac{\mathrm{d}x}{a^2+x^2}=\frac{1}{a}\operatorname{arctg}\frac{x}{a}+C \quad (a>0)$$